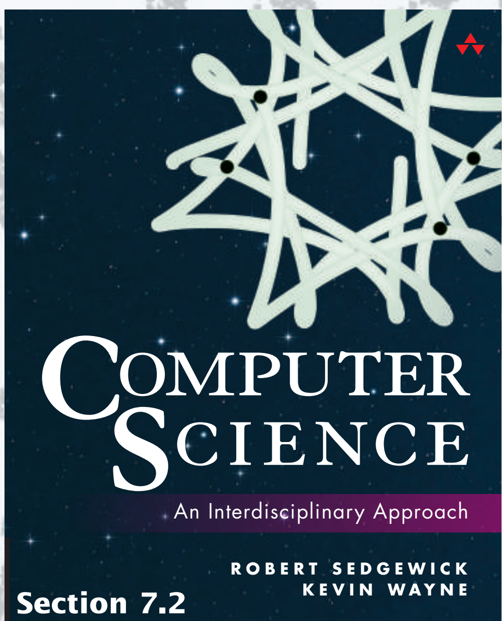



http://introcs.cs.princeton.edu

# 14. Introduction to Theoretical CS

# 14. Introduction to Theoretical CS

- Overview
- Regular expressions
- DFAs
- Applications
- Limitations

# Introduction to theoretical computer science

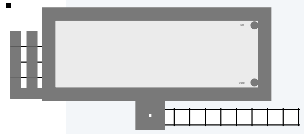
## Fundamental questions

- What can a computer do?
- What can a computer do with limited resources?

## General approach

- Don't talk about specific machines or problems.
- Consider minimal abstract machines.
- Consider general classes of problems.




Surprising outcome. Sweeping and relevant statements about *all* computers.

## Why study theory?

In theory...

- Deeper understanding of computation.
- Foundation of all modern computers.
- Pure science.
- Philosophical implications.

In practice...

- Web search: theory of pattern matching.
- Sequential circuits: theory of finite state automata.
- Compilers: theory of context free grammars.
- Cryptography: theory of computational complexity.
- Data compression: theory of information.
- ...

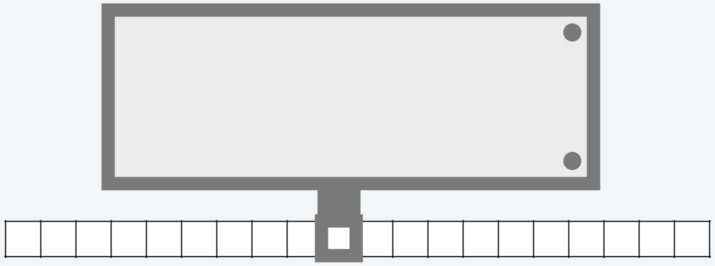

*" In theory there is no difference between theory and*

*— Yogi Berra*

# Abstract machines

## Abstract machine

- Mathematical model of computation.
- Each machine defined by specific rules for transforming input to output.
- This lecture: Deterministic finite automata (DFAs).



## Formal language

- A set of strings.
- Each defined by specific rules that characterize it.
- This lecture: Regular expressions (REs).

```
madam im adam
a man a plan a canal panama
able i was ere i saw elba
evil olive
go hang a salami im a lasagna hog
pull up if i pull up
...
```

## Questions for this lecture

- Is a given string in the language defined by a given RE, or not?
- Can a DFA help answer this question?

CS.17.A.Theory.Overview

COMPUTER SCIENCE
S E D G E W I C K / W A Y N E

PART II: ALGORITHMS, MACHINES, and THEORY

# 14. Introduction to Theoretical CS

- Overview
- **Regular expressions**
- DFAs
- Applications
- Limitations

## Pattern matching

Pattern matching problem. Is a given string an element of a given set of strings?

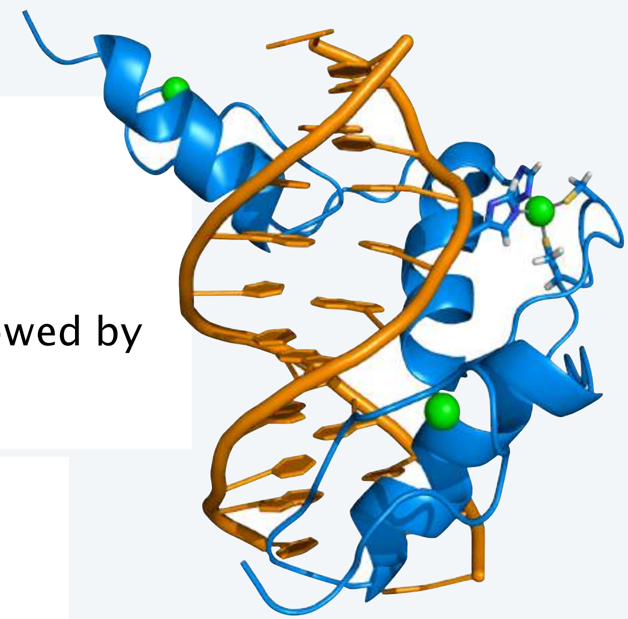
Example 1 (from computational biochemistry)

An amino acid is represented by one of the characters `CAVLIMCRKHDENQSTYFWP`.

A protein is a string of amino acids.

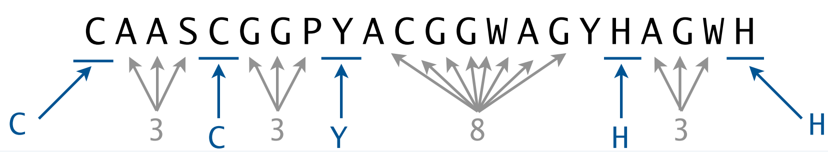
A $C_2H_2$-type zinc finger domain signature is

- C followed by 2, 3, or 4 amino acids, followed by
- C followed by 3 amino acids, followed by
- L, I, V, M, F, Y, W, C, or X followed by 8 amino acids, followed by
- H followed by 3, 4, or 5 amino acids, followed by H.



Q. Is this protein in the $C_2H_2$-type zinc finger domain?

A. Yes.



# Pattern matching

Example 2 (from commercial computing)

An e-mail address is

- A sequence of letters, followed by
- the character "@", followed by
- followed by a nonempty sequence of lowercase letters, followed by the character "."
- [any number of occurrences of the previous pattern]
- "edu" or "com" (others omitted for brevity).

Q. Which of the following are e-mail addresses?

| | A. |
|---|---|
| rs@cs.princeton.edu | ✓ |
| not an e-mail address | ✗ |
| wayne@cs.princeton.edu | ✓ |
| eve@airport | ✗ |
| rs123@princeton.edu | ✗ |

Oops, need to fix description →

Challenge. Develop a precise description of the set of strings that are legal e-mail addresses.

## Pattern matching

Example 3 (from genomics)

A nucleic acid is represented by one of the letters a, c, `t`, or g.

A genome is a string of nucleic acids.

A Fragile X Syndrome pattern is a genome having an occurrence of gcg, followed by any number of cgg or agg triplets, followed by `ctg`.

Note. The number of triplets correlates with Fragile X Syndrome, a common cause of mental retardation.

Q. Does this genome contain a such a pattern?

`gcggcgtgtgtgcgagagagtgggtttaaagctg|gcg|cgg|agg|cgg|ctg|gcgcggaggctg`

A. Yes.

`gcgcggaggcggctg`

gcg start mark

sequence of cgg and agg triplets

ctg end mark

# Regular expressions

A regular expression (RE) is a notation for specifying a set of strings ( a formal language).

An RE is either

- The empty set
- The empty string
- A single character or wildcard symbol
- An RE enclosed in parentheses
- The *concatenation* of two or more REs
- The *union* of two or more REs
- The *closure* of an RE
  (any number of occurrences)

| *operation* | *example RE* | *matches* (IN *the set*) | *does not match* (NOT *in the set*) |
|---|---|---|---|
| *concatenation* | `aabaab` | `aabaab` | *every other string* |
| *wildcard* | `.u.u.u.` | `cumulus`<br>`jugulum` | `succubus`<br>`tumultuous` |
| *union* | `aa \| baab` | `aa`<br>`baab` | *every other string* |
| *closure* | `ab*a` | `aa`<br>`abbba` | `ab`<br>`ababa` |
| *parentheses* | `a(a\|b)aab` | `aaaab`<br>`abaab` | *every other string* |
| | `(ab)*a` | `a`<br>`ababababa` | `aa`<br>`abbba` |

## More examples of regular expressions

The notation is surprisingly expressive.

| *regular expression* | *matches* | *does not match* |
| --- | --- | --- |
| `.*spb.*`<br>*contains the trigraph* spb | raspberry<br>crispbread | subspace<br>subspecies |
| `a* \| (a*ba*ba*ba*)*`<br>*multiple of three* b's | bbb<br>aaa<br>bbbaababbaa | b<br>bb<br>baabbbaa |
| `.*0....`<br>*fifth to last digit is* 0 | 1000234<br>98701234 | 111111111<br>403982772 |
| `.*gcg(cgg\|agg)*ctg.*`<br>*fragile X syndrome pattern* | ...gcgctg...<br>...gcgcggctg...<br>...gcgcggaggctg... | gcgcgg<br>cggcggcggctg<br>gcgcaggctg |

## Generalized regular expressions

Additional operations further extend the utility of REs.

| *operation* | *example RE* | *matches* | *does not match* |
|---|---|---|---|
| one or more | `a(bc)+de` | abcde<br>abcbcde | ade<br>bcde |
| character class | `[A-Za-z][a-z]*` | lowercase<br>Capitalized | camelCase<br>4illegal |
| exactly j | `[0-9]{5}-[0-9]{4}` | 08540-1321<br>19072-5541 | 111111111<br>166-54-1111 |
| between j and k | `a.{2,4}b` | abcb<br>abcbcb | ab<br>aaaaaab |
| negation | `[^aeiou]{6}` | rhythm | decade |
| whitespace | `\s` | *any whitespace char*<br>(*space, tab, newline...*) | *every other character* |

Note. These operations are all *shorthand*.
They are very useful but not essential.

RE: `(a|b|c|d|e)(a|b|c|d|e)*`
shorthand: `(a-e)+`

# Example of describing a pattern with a generalized RE

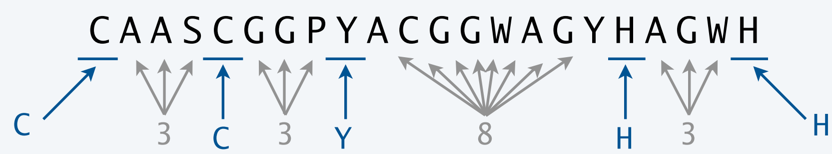
A $C_2H_2$-type zinc finger domain signature is

- C followed by 2, 3, or 4 amino acids, followed by
- C followed by 3 amino acids, followed by
- L, I, V, M, F, Y, W, C, or X followed by 8 amino acids, followed by
- H followed by 3, 4, or 5 amino acids, followed by

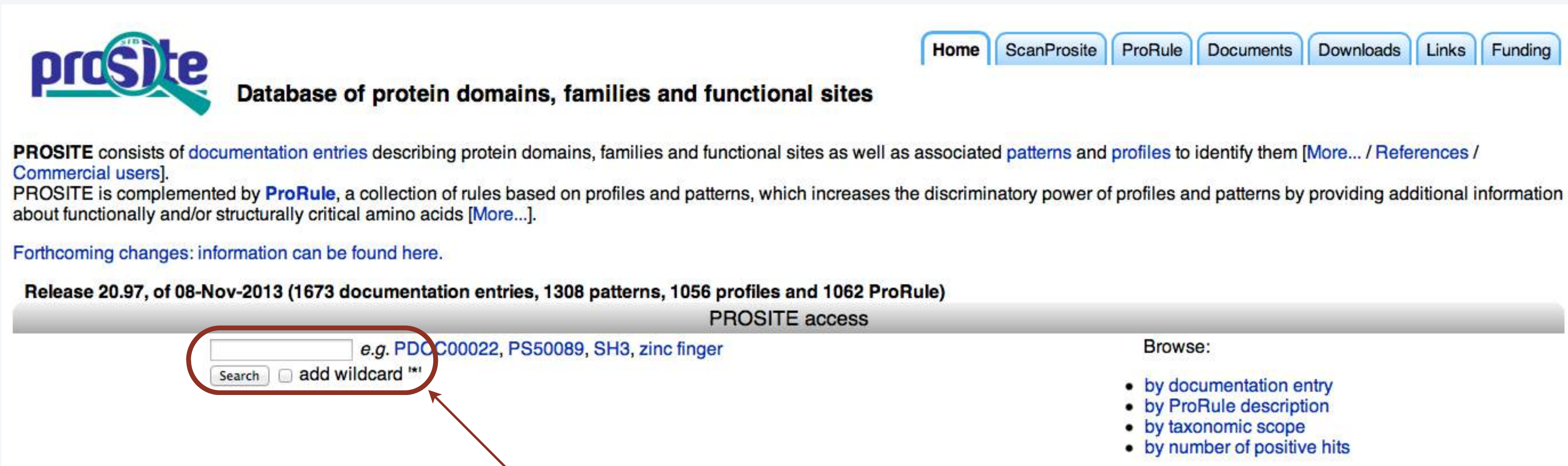
Q. Give a generalized RE for all such signatures.

A. `C.{2,4}C...[LIVMFYWCX].{8}H.{3,5}H`

"Wildcard" matches any of the letters
CAVLIMCRKHDENQSTYFWP



# Example of a real-world RE application: PROSITE




Type an RE here

# Another example of describing a pattern with a generalized RE

An e-mail address is

- A sequence of letters, followed by
- the character "@", followed by
- the character "." , followed by a nonempty sequence of lowercase letters, followed by
- [any number of occurrences of the previous pattern]
- "edu" or "com" (others omitted for brevity).

Q. Give a generalized RE for e-mail addresses.

A. `[a-z]+@([a-z]+\.)+(edu|com)`

Exercise. Extend to handle `rs123@princeton.edu`, more suffixes such as `.org`, and any other extensions you can think of.

Next. Determining whether a given string matches a given RE.

## Pop quiz 1 on REs

Q. Which of the following strings match the RE `a*bb(ab|ba)*` ?

is in the set it describes

1. abb
2. aaba
3. abba
4. bbbaab
5. cbb
6. bbababbab

## Pop quiz 2 on REs

Q. Give an RE for *genes*

- Characters are a, c, t or g.
- Starts with atg (a *start codon*).
- Length is a multiple of 3.
- Ends with tag, taa, or ttg (a *stop codon*).

*Image sources*

http://en.wikipedia.org/wiki/Homology_modeling#/media/File:DHRS7B_homology_model.png

# 14. Introduction to Theoretical CS

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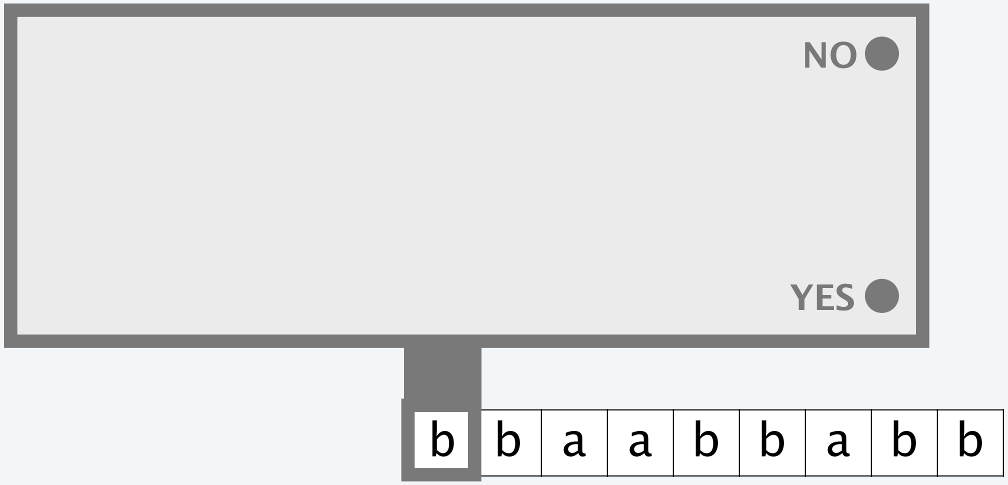
- **DFAs**
- Applications
- Limitations

## Deterministic finite automata (DFA)

A DFA is an abstract machine that solves a pattern matching problem.

- A string is specified on an input tape (no limit on its length).
- The DFA reads each character on input tape once, moving left to right.
- The DFA lights "YES" if it *recognizes* the string, "NO" otherwise.

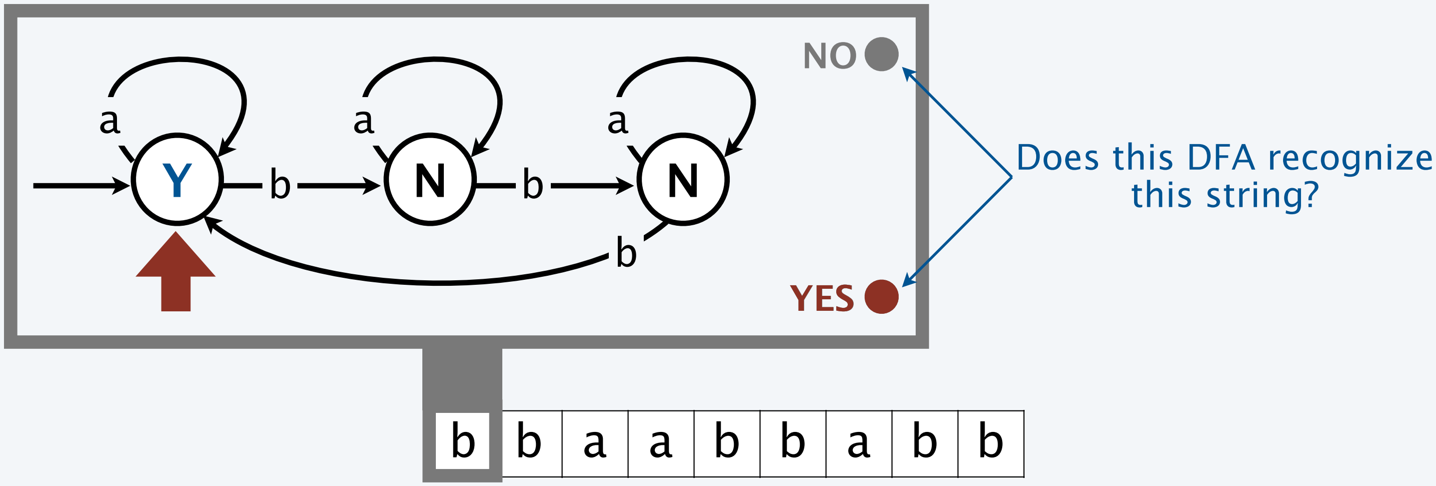
Each DFA defines a *language* (the set of strings that it recognizes).



# Deterministic finite automata details and example

A DFA is an abstract machine with a finite number *states*, each labeled Y or N, and *transitions* between states, each labeled with a symbol. One state is the *start* state.

- Begin in the *start* state.
- Read an input symbol and move to the indicated state.
- Repeat until the last input symbol has been read.
- Turn on the "YES" or "NO" light according to the label on the current state.



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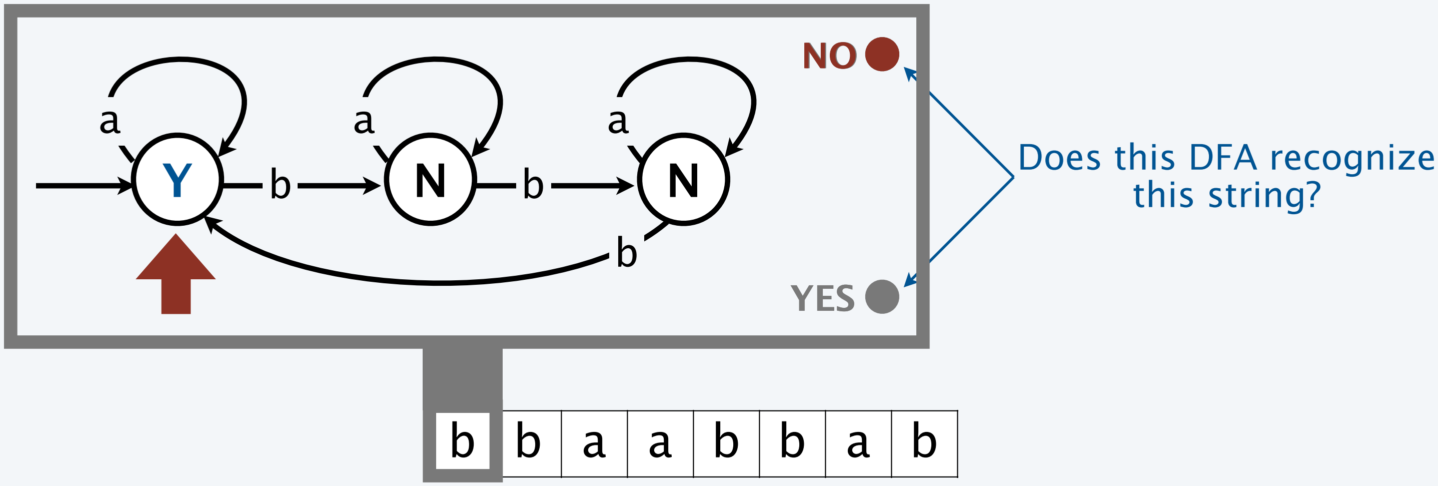
- Begin in the *start* state.
- Read an input symbol and move to the indicated state.
- Repeat until the last input symbol has been read.
- Turn on the "YES" or "NO" light according to the label on the current state.



# Simulating the operation of a DFA

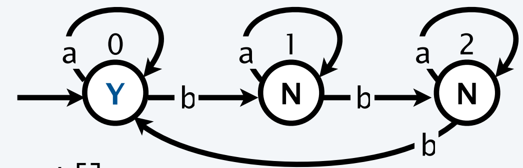
```
public class DFA
{
   private int start;
   private boolean[] action;
   private ST<Character, Integer>[] next;

   public DFA(String filename)
   {  /* Fill in data structures */  }

   public boolean recognizes(String input)
   {
      int state = start;
      for (int i = 0; i < input.length(); i++)
         state = next[state].get(input.charAt(i));
      return action[state];
   }

   public static void main(String[] args)
   {
      DFA dfa = new DFA(args[0]);
      while (!StdIn.isEmpty())
      {
         input = StdIn.readString();
         if (dfa.recognizes(input)) StdOut.println("Yes");
         else                       StdOut.println("No");
      }
   }
}
```

symbol table to map chars a, b, ... to next state 0, 1, ...




action[]   next[]

| | | | a | b |
|---|---|---|---|---|
| 0 | True | 0 | 0 | 1 |
| 1 | False | 1 | 1 | 2 |
| 2 | False | 2 | 2 | 0 |

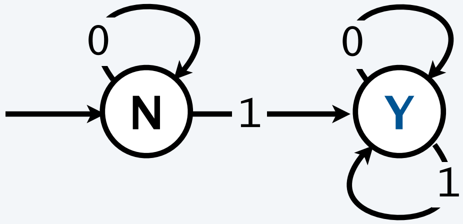
# states → 3
alphabet → ab
start state → 0

```
% more b3.txt
3
ab
0
True   0 1
False  1 2
False  2 0

% java DFA b3.txt
bababa
Yes
bb
No
abbabbababbbabaaa
Yes
abbabbababbba
No
```

## Pop quiz 1 on DFAs

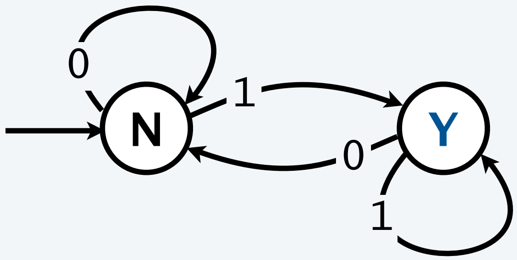
Q. Which of the following strings does this DFA accept?




1. Bitstrings that end in 1
2. Bitstrings with an equal number of occurrences of 01 and 10
3. Bitstrings with more 1s than 0s
4. Bitstrings with an equal number of occurrences of 0 and 1
5. Bitstrings with at least one 1

## Pop quiz 2 on DFAs

Q. Which of the following strings does this DFA accept?

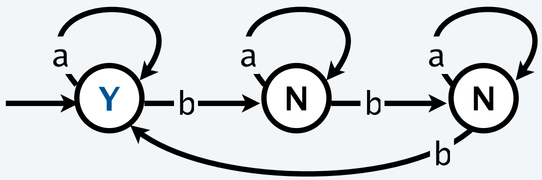



1. Bitstrings with at least one 1
2. Bitstrings with an equal number of occurrences of 01 and 10
3. Bitstrings with more 1s than 0s
4. Bitstrings with an equal number of occurrences of 0 and 1
5. Bitstrings that end in 1

# Kleene's theorem

S ≡ the set of ab strings where the number of occurrences of b is a multiple of 3

DFA for S




RE for S a* | (a*ba*ba*ba*)*

### Two ways to define a set of strings (language)

- Regular expressions (REs).
- Deterministic finite automata (DFAs).

Remarkable fact. DFAs and REs are *equivalent*.

### Equivalence theorem (Kleene)

Given any RE, there exists a DFA that accepts the same set of strings.

Given any DFA, there exists an RE that matches the same set of strings.

Steven Kleene
1909–1994

### Consequence: A way to solve the RE pattern matching problem

- Build the DFA corresponding to the given RE.
- Simulate the operation of the DFA.

*Image sources*

http://math.library.wisc.edu/images/skleene.gif

# 14. Introduction to Theoretical CS

- Overview
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- **Applications**
- Limitations

# GREP: a solution to the RE pattern matching problem

"GREP" (Generalized Regular Expression Pattern matcher).

- Developed by Ken Thompson, who designed and implemented Unix.
- Indispensable programming tool for decades.
- Found in most development environments, including Java.

Practical difficulty: The DFA might have *exponentially* many states.


Interested in details? Take a course in algorithms.

A more efficient algorithm: use Nondeterministic Finite Automata (NFA)

- Build the NFA corresponding to the given RE.
- Simulate the operation of the NFA.


Ken Thompson
1983 Turing Award

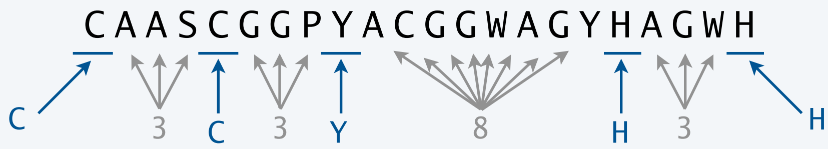
# REs in Java

Java's String class implements GREP.

| public class String | |
|---|---|
| ... | |
| boolean matches(String re) | *does this string match the given RE?* |
| ... | |

```
String re = "C.{2,4}C...[LIVMFYWC].{8}H.{3,5}H";
String zincFinger = "CAASCGGPYACGGAAGYHAGAH";
boolean test = zincFinger.matches(re);
```

true!



# Java RE client example: Validation

```
public class Validate
{
   public static void main(String[] args)
   {
      String re = args[0];
      while (!StdIn.isEmpty())
      {
         String input = StdIn.readString();
         StdOut.println(input.matches(re));
      }
   }
}
```

Does a given string match a given RE?

- Take RE from command line.
- Take strings from StdIn.

need quotes to "escape" the shell

```
% java Validate "C.{2,4}C...[LIVMFYWC].{8}H.{3,5}H"
CAASCGGPYACGGAAGYHAGAH
true
CAASCGGPYACGGAAGYHGAH
false
```

$C_2H_2$ type zinc finger domain

```
% java Validate "[$_A-Za-z][$_A-Za-z0-9]*"
ident123
true
123ident
false
```

legal Java identifier

```
% java Validate "[a-z]+@([a-z]+\.)+(edu|com)"
wayne@cs.princeton.edu
true
eve@airport
false
```

valid email address (simplified)

Applications

- Scientific research.
- Compilers and interpreters.
- Internet commerce.
- ...

## Beyond matching

Java's String class contains other useful RE-related methods.

- RE search and replace
- RE delimited parsing

| public class String | |
|---|---|
| ... | |
| String replaceAll(String re, String to) | *replace all occurrences of substrings matching RE with to* |
| String[] split(String re) | *split the string around matches of the given RE* |
| ... | |

Tricky notation (typical in string processing): \ signals "special character" so "\\\\" means "\\" and "\\\\s" means "\\s"

Examples using the RE "\\\\s+" (matches one or more whitespace characters).

Replace each sequence of at least one whitespace character with a single space.

```
String s = StdIn.readAll();
s = s.replaceAll("\\s+", " ");
```

Create an array of the words in StdIn (basis for StdIn.readAllStrings() method)

```
String s = StdIn.readAll();
String[] words = s.split("\\s+");
```

# Way beyond matching

Java's `Pattern` and `Matcher` classes give fine control over the GREP implementation.

| `public class Pattern` | |
|---|---|
| ... | |
| `static Pattern compile(String re)` | *parse the* `re` *to construct a* `Pattern` |
| `Matcher matcher(String input)` | *create a* `Matcher` *that can find substrings matching the pattern in the given input string* |
| ... | |

Why not a constructor?
Good question.

| `public class Matcher` | |
|---|---|
| ... | |
| `boolean find()` | *set internal variable* `match` *to the next substring that matches the RE in the input. If none, return* `false`*, else return* `true` |
| `String group()` | *return* `match` |
| `String group(int k)` | *return the kth group (identified by parens within RE) in* `match` |
| ... | |

[A sophisticated interface designed for pros, but very useful for everyone.]

# Java pattern matcher client example: Harvester

```
import java.util.regex.Pattern;
import java.util.regex.Matcher;

public class Harvester
{
   public static void main(String[] args)
   {
      String re       = args[0];
      In in           = new In(args[1]);
      String input    = in.readAll();
      Pattern pattern = Pattern.compile(re);
      Matcher matcher = pattern.matcher(input);
      while (matcher.find())
         StdOut.println(matcher.group());

   }
}
```

Harvest information from input stream

- Take RE from command line.
- Take input from file or web page.
- Print all substrings matching RE.

```
% java Harvester "gcg(cgg|agg)*ctg" chromosomeX.txt
gcgcggcggcggcggcggctg
gcgctg
gcgctg
gcgcggcggcggaggcggaggcggctg

% java Harvester "[a-z]+@([a-z]+\.)+(edu|com)" http://www.cs.princeton.edu/people/faculty
...
rs@cs.princeton.edu
...
wayne@cs.princeton.edu
...
```

harvest patterns from DNA

harvest email addresses from web for spam campaign.
(no email addresses on that site any more)

## Applications of REs

Pattern matching and beyond.

- Compile a Java program.
- Scan for virus signatures.
- Crawl and index the Web.
- Process natural language.
- Access information in digital libraries.
- Search-and-replace in a word processors.
- Process NCBI and other scientific data files.
- Filter text (spam, NetNanny, ads, Carnivore, malware).
- Validate data-entry fields (dates, email, URL, credit card).
- Search for markers in human genome using PROSITE patterns.
- Automatically create Java documentation from Javadoc comments.

GREP and related facilities are built in to Java, Unix shell, PERL, Python ...

virtually *every* computing environment

*Image sources*

http://en.wikipedia.org/wiki/Ken_Thompson#/media/File:Ken_n_dennis.jpg

# 14. Introduction to Theoretical CS

- Overview
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- DFAs
- Applications
- **Limitations**

## Summary

### Programmers

- Regular expressions are a powerful pattern matching tool.
- Equivalent DFA/NFA paradigm facilitates implementation.
- Combination greatly facilitates real-world string data

### Theoreticians

- REs provide compact descriptions of sets of strings.
- DFAs are abstract machines with equivalent descriptive power.
- Are there languages and machines with more descriptive power?

### You

- CS core principles provide useful tools that you can exploit now.
- REs and DFAs provide an introduction to theoretical CS.

## Basic questions

Q. Are there sets of strings that cannot be described by *any* RE?

A. Yes.

- Bitstrings with equal number of 0s and 1s (stay tuned).
- Strings that represent legal REs.
- Decimal strings that represent prime numbers.
- DNA strings that are Watson-Crick complemented palindromes.
- ...

Q. Are there sets of strings that cannot be described by *any* DFA?

A. Yes.

- Bit strings with equal number of 0s and 1s (see next slide).
- Strings that represent legal REs.
- Decimal strings that represent prime numbers.
- DNA strings that are Watson-Crick complemented palindromes.
- ...

The *same* question, by Kleene's theorem

# A limit on the power of REs and DFAs

Proposition. There exists a set of strings that cannot be described by any RE or DFA.

Proof sketch. No DFA can recognize the set of bitstrings with equal number of 0s and 1s.

- *Assume that you have such a DFA*, with $N$ states.
- It recognizes the string with $N+1$ 0s followed by $N+1$ 1s.
- Some state is *revisited* when scanning the 0s in that string.
- Delete the substring of 0s between visits of that state.
- DFA recognizes that string, too.
- It does not have equal number of 0s and 1s.
- *Proof by contradiction*: the assumption that such a DFA exists must be false.

Ex. $N = 10$

| | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 3 | 5 | 9 | 8 | 7 | 5 | . | . | . | | | | | | | | | | | | |

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 3 | 5 | . | . | . | | | | | | | | | | | | |

## Another basic question

can recognize more sets of strings

Q. Are there abstract machines that are more powerful than DFAs?

A. Yes. A 1-stack DFA can recognize

- Bitstrings with equal number of 0s and 1s.
- Strings that represent legal REs.

Proof. [details omitted]

## Yet another basic question

Q. Are there abstract machines that are more powerful than a 1-stack DFA?

A. Yes. A 2-stack DFA can recognize

- Decimal strings that represent prime numbers.
- Strings that represent legal Java programs.
- ...

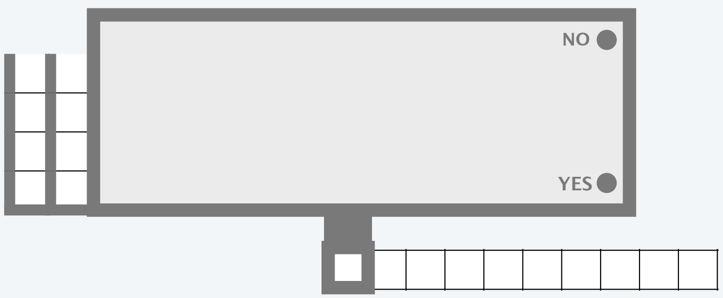
[stay tuned for next lecture]

# One last basic question

Q. Are there machines that are more powerful than a 2-stack DFA?

A. No! Not even a roomful of supercomputers (!!!)

[stay tuned for next lecture]




*two machines with equal computational power*

*Image sources*

https://openclipart.org/detail/211418/thenanobel-programming

COMPUTER SCIENCE

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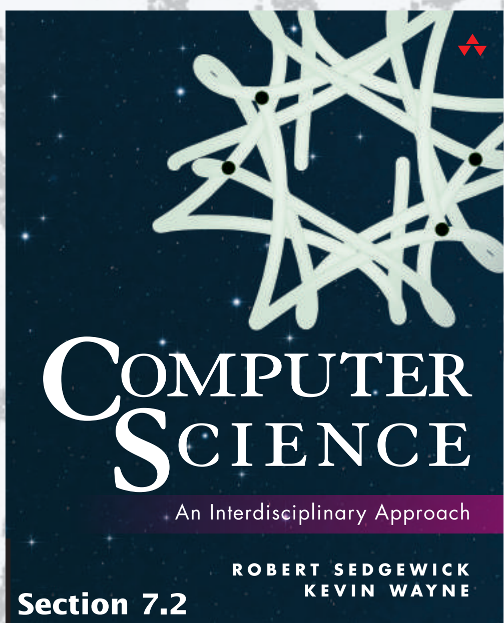
PART II: ALGORITHMS, MACHINES, and THEORY




http://introcs.cs.princeton.edu

# 14. Introduction to Theoretical CS